Solutions

MA114 Summer 2018
Worksheet 6a – Improper Integrals II – 6/18/18

1. Determine if the following integrals are proper or improper. Do not evaluate any of the integrals.

(a) $\displaystyle\int_0^2 \frac{x\,dx}{x^2-5x+6}$ Improper, asymptote at $x=2$

(b) $\displaystyle\int_1^2 \frac{dx}{2x-1}$ Proper, cts on $[1,2]$

(c) $\displaystyle\int_1^2 \ln(t-1)\,dt$ Improper, asymptote at $t=1$

(d) $\displaystyle\int_{-\infty}^{\infty} \frac{\sin(x)}{1+x^2}\,dx$ Improper, infinite bounds

2. Evaluate $\displaystyle\int_0^2 \frac{1}{\sqrt{x}}\,dx$. asymptote at $x=0$:

$$= \lim_{R\to 0^+} \int_R^2 x^{-1/2}\,dx = \lim_{R\to 0^+} 2x^{1/2}\Big|_R^2$$
$$= \lim_{R\to 0^+} 2\sqrt{2} - 2\sqrt{R}$$
$$= \boxed{2\sqrt{2}}$$

3. Use the Comparison Theorem to determine whether the following integrals converge or diverge.

(a) $\displaystyle\int_1^{\infty} \frac{2+e^{-x}}{x}\,dx$

$\frac{2+e^{-x}}{x} > \frac{2}{x} > \frac{1}{x}$ for $x \geq 1$ and we know $\int_1^\infty \frac{1}{x}\,dx$ diverges (p-integral with $p=1$)

So by the Comparison Theorem, $\int_1^\infty \frac{2+e^{-x}}{x}\,dx$ also diverges.

(b) $\displaystyle\int_1^{\infty} \frac{x+1}{\sqrt{x^6+x}}\,dx$

$\frac{x+1}{\sqrt{x^6+x}} \approx \frac{x}{\sqrt{x^6}} = \frac{1}{x^2}$, so we guess it converges.

$$\frac{x+1}{\sqrt{x^6+x}} < \frac{x}{\sqrt{x^6+x}} = \frac{\sqrt{x^2}}{\sqrt{x^6+x}} = \sqrt{\frac{1}{x^4+\frac{1}{x}}}$$
$$< \sqrt{\frac{1}{2x^4}}$$
$$= \frac{1}{\sqrt{2}}\frac{1}{x^2}$$

$x^4 > \frac{1}{x}$ for $x>1$, so $\frac{1}{x^4+\frac{1}{x}} < \frac{1}{2x^4}$

Now $\frac{1}{\sqrt{2}}\int_1^\infty \frac{1}{x^2}\,dx$ converges (p-integral), so by the Comp. Thm, $\int_1^\infty \frac{x+1}{\sqrt{x^6+x}}\,dx$ also converges.